

Solution to HWE problem considered in class

Suppose we denote

$$\text{Prob}(AA) = P_1$$

$$\text{Prob}(AB) = 2Q_1$$

$$\text{Prob}(BB) = R_1 \quad (1)$$

Then we have seen as a first expression of HWE:

$$\text{Prob}(AA) = p_1^2$$

$$\text{Prob}(AB) = 2 p_1 q_1$$

$$\text{Prob}(BB) = q_1^2 \quad (2)$$

where $p_1 = \text{Prob}(A)$, $q_1 = \text{Prob}(B) = 1 - \text{Prob}(A)$

An alternative way of stating the HWE property is

$$Q_1^2 = P_1 R_1 \quad (3)$$

The question was: What is the connection between expression (3) and the equalities in (2)?

Answer:

Using both expressions (1) and (2),

$$p_1 = \sqrt{P_1}, \quad q_1 = \sqrt{R_1}$$

Therefore, $2 p_1 q_1 = 2 \sqrt{P_1} \sqrt{R_1}$, which is equal to $2Q_1$ since both $2 p_1 q_1$ and $2Q_1$ refer to $\text{Prob}(AB)$.

In other words, after squaring, $Q_1^2 = P_1 R_1$ indeed.